Allan Variance Analysis of Random Noise Modes in Gyroscopes

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Bias Instability (BI) refers to the additive error in a gyroscope's output with stochastic (random) characteristics. Different random noise modes (i.e. stochastic processes with different properties) dominate the overall gyroscope's noise level depending on the averaging time, which in turn defines the useful sensor bandwidth. The *Allan variance* method is often used to identify and quantify random noise modes with different autocorrelation properties and expose their effect when the output signal is averaged or integrated over time. *Allan variance* analysis of a time domain signal $w(t)$ consists of computing its root *Allan variance (RAVAR)* $\sigma(\tau)$ for different integration time constants τ and then analyzing the characteristic regions and log-log scale slopes of the $\sigma(\tau)$ curve to identify different noise modes, i.e., random components of the signal with different autocorrelation power laws.

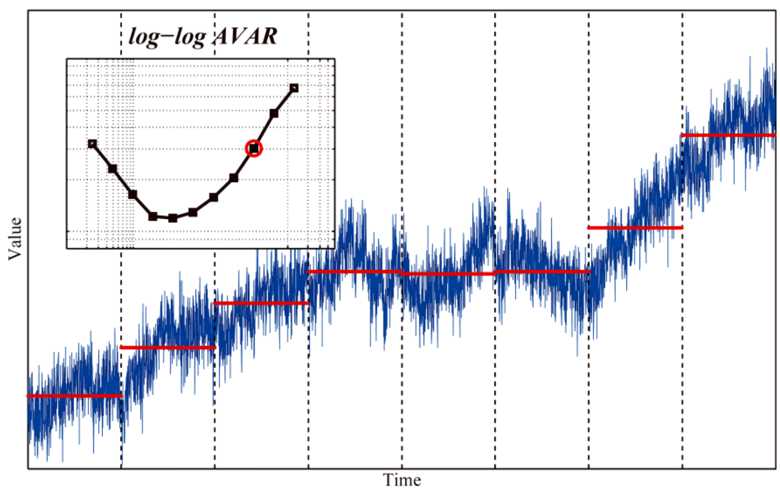

Figure 1: Illustration of the Allan variance analysis procedure using a simulated random process with a white noise (-1/2 slope in the RAVAR plot, $1/f^0$ PSD) and a random walk component (integral of white noise, +1/2 slope in the RAVAR plot, $1/f^2$ PSD).

The first step of *Allan variance* analysis is to acquire a time history $w(t)$ of the gyroscope's output using an experimental setup or a numerical simulation. The second step is to fix the integration time constant τ, divide the time history of the signal into K number of clusters of width τ, then average the signal over each cluster to obtain $w_{i=1...K}$, and finally compute the *Allan variance* defined as one half of the average of the squares of the differences between the successive averaged values: $\sigma^2(\tau)=0.5<(w_{i+1}-w_i)^2>$. The described sequence of steps yields the estimated value of the root *Allan variance* $\sigma(\tau)$ for the chosen integration time constant τ, ***Figure 1***. To obtain the whole $\sigma(\tau)$ curve, the computation is repeated multiple times for a sequence of τ values. Typically, the integration time values τ are iterated through the multiples or powers of the time discretization step (inverse of the signal sampling rate). While *an Allan variance* curve for a signal contains essentially the same information as the signal's power spectral density (PSD), it presents the information in an alternative and often more convenient way. The power laws of PSD and RAVAR are related to each other in the following way:

Equation 1 $\quad PSD(f) \propto f^{\alpha}$ ***is equivalent to,*** $\sigma(\tau) \propto \tau^{\beta}$,

where $\beta = -(\alpha+1)/2$ ***and*** $f=1/\tau$.

Table 1 presents a classification of the three main noise components (*ARW*, flicker, *ARRW*) in MEMS gyroscopes based on their power spectral density and *Allan variance* power laws.

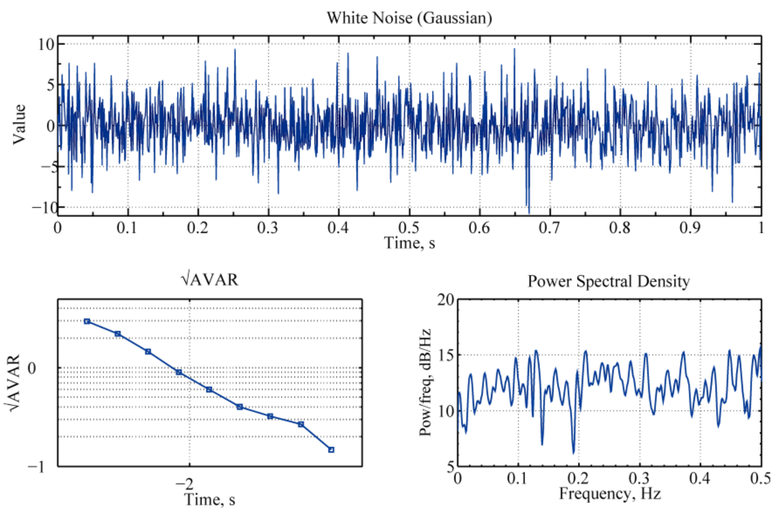

Figure 2: Illustration of the $1/f^{0}$ white noise properties in time, frequency, and root Allan variance ($\tau^{-1/2}$) domains using a numerical example.

White noise is a random signal with a constant power spectral density. While this definition presents a mathematical abstraction not physically possible due to its infinite total power, it gives a useful model of random processes with very short characteristic autocorrelation times. For example, the random motion of particles due to the mechanical thermal noise at room temperature has a characteristic cutoff frequency of approximately 6 THz. Important properties of a *white noise* type random process in time, frequency, and root *Allan variance* domains are shown in ***Figure 2*** using a computer simulated signal. Power spectral density of the *white noise* is independent of frequency, i.e. $PSD(f) \propto f^{\alpha}$ with $\alpha = 0$. As expected from ***Equation 1***, the numerically estimated root *Allan variance* of the *white noise* sample is governed by $\sigma(\tau) \propto \tau^{\beta}$ with $\beta = -1/2$. In the log-log root *Allan variance* plot, the *white noise* is identified by fitting the $-1/2$ sloped part of the curve with $y = ax^{-1/2}$ and estimating the value of the coefficient *a*. In rate gyroscopes, the *white noise* is often referred to as the *rate resolution* and is quantified in terms of its power spectral density in deg/s/√Hz or deg/hour/√Hz units. When integrated over time to obtain the angular orientation, *white noise* of rate becomes a *random walk* of angle ($1/f^{2}$ random process), which is called *Angle Random Walk (ARW)* and expressed in deg/√hour units.

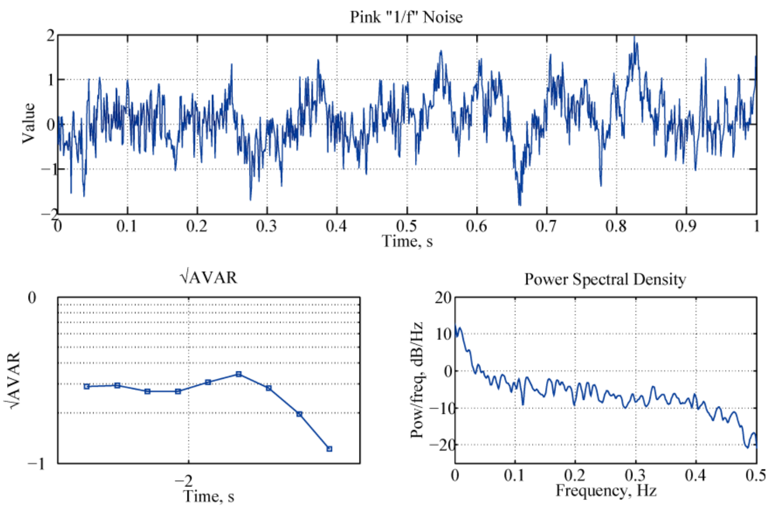

Figure 3 Illustration of the 1/f ("pink") noise properties in time, frequency, and root Allan variance (τ^0) domains using a numerical example.

Random processes with *1/f* PSD are naturally the next class of noise to consider. *Flicker noise*, commonly encountered in electronic circuits, falls into this category. *Figure **3*** illustrates time, frequency, and *Allan variance* domain properties of *1/f* (or "pink") noise using a computer simulation. A pink *1/f* noise can be obtained by low-pass filtering a *white noise* with a -3 dB per octave filter. Pink noise can be distinguished from a *white noise* by more visible low frequency fluctuations. As expected from ***Equation 1***, RAVAR has essentially a flat profile (τ^{0}) in the case of pink *1/f* noise. In rate gyroscopes, *1/f* noise often defines the smallest value of the root *Allan variance*, which is referred to as the *Bias Instability* and has units of deg/s or deg/hour.

The third important type of random noise is "red" noise characterized by $1/f^2$ PSD. This type of random process is called a *random walk* and occurs whenever a *white noise* is integrated over time. While not correct from the color spectral parallel, red $1/f^2$ noise is sometimes referred to as Brown noise, in honor of Robert Brown who first studied Brownian motion. ***Figure 4*** illustrates time, frequency, and *Allan variance* domain properties of $1/f^2$ *random walk* process obtained by numerical integration of a simulated *white noise* signal. Another way to obtain a *random walk* process is to filter a *white noise* signal with a -6 dB per octave filter. *Random walk* signals show strong low frequency fluctuations. The power spectral density of a *random walk* is given by PSD(f) $\propto f^{\alpha}$ with $\alpha = -2$ and its root *Allan variance* profile is governed by $\sigma(\tau) \propto \tau^{\beta}$ with $\beta = +1/2$. In a log-log root *Allan variance* plot, the *random walk* component is identified by fitting the *+1/2* sloped part of the curve with $y = bx^{+1/2}$ and estimating the value of the coefficient *b*. In rate gyroscopes, $1/f^2$ noise is typically referred to as the *Angle Rate Random Walk (ARRW)* and quantified in deg/s*√Hz, deg/hour*√Hz, or, equivalently deg*√hour units.

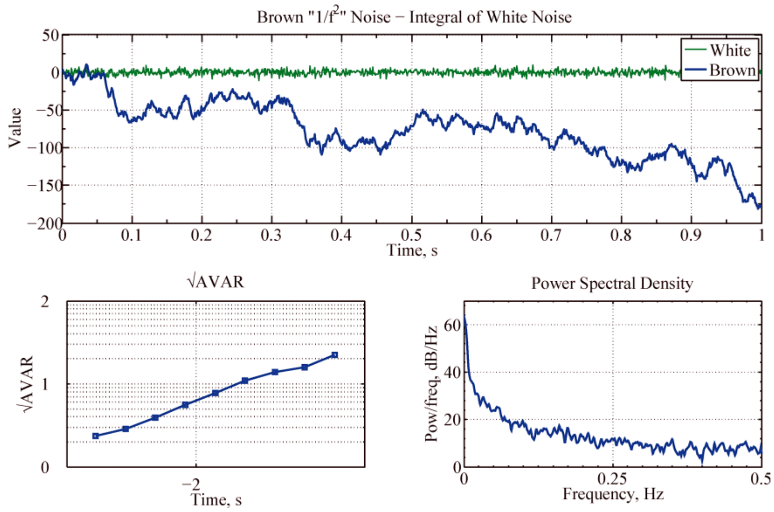

Figure 4 Illustration of the $1/f^{2}$ ("red" or "Brown") noise properties in time, frequency, and root Allan variance ($\tau^{+1/2}$) domains using numerical example.

When a rate gyroscope is used to track or maintain the angular orientation of an object, the rate signal is integrated over time together with the associated *white noise* of rate, flicker noise of rate, and *random walk* of rate. Upon integration, the three rate noise sources with $1/f^{0}$, $1/f^{+1}$, and $1/f^{+2}$ densities produce drifts of angle with $1/f^{+2}$, $1/f^{+3}$, and $1/f^{+4}$ power spectral densities, respectively. Each of these angle errors grows fast with time, leading to degradation of orientation precision. Angle measuring (rate integrating) micromachined gyroscopes can potentially provide much better precision since the time integration of the noise signals is avoided.

Table 1 Classification of three main random noise modes as applied to rate gyroscopes.

Spectral type	Example sources	PSD(*f*) power law f^{α}	$\sigma(\tau)$ power law τ^{β}	Averaging over time	Associated gyroscope parameters
White	Johnson-Nyquist thermal noise	f^{0}	$\tau^{-1/2}$	Good	Rate resolution in deg/s/√Hz, deg/hour/√Hz
					Angle Random Walk (ARW) in deg/√hour
Pink	Electronics flicker	f^{-1}	τ^{0}	Neutral	Flicker, Bias Instability in deg/s, deg/hour
Red (Brown)	White noise accumulation	f^{-2}	$\tau^{+1/2}$	Bad	Angle Rate Random Walk (ARRW) in deg/s*√Hz, deg/hour*√Hz, deg*√hour